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Assignment 10: MTH 213, Fall 2017

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QUESTION 1. Assume there are 1205 persons in a gathering. Then

(i) There are at least n persons who were born in the same month and on the same day. What is the maximum value of n that we all are sure about?

(ii) Assume that all 1205 persons were born between 1998 and 2000. Then there are at least n persons who were born in the same year and in the same month and on the same day. What is the maximum value of n that we all are sure about?

QUESTION 2. We have 6 holes labeled from 1 to 6 and we have 3 balls (red, blue, green). We need to put each ball in one hole. Find all possible ways? [Answer 6C3 X 3P3)

QUESTION 3. Assume that a class has 10 males and 7 females. We need to form a committee with 6 persons.

(i) In how many ways can we form a committee with at least 2 males are included? (Answer 10C2 X 7C4 + 10C3X7C3 + 10C4X7C2 + 10C5 X 7C1 + 10C6. Another solution:

17C6 - (10C1 X 7C5 + 10C0 X 7C6))

(ii) In how many ways can we form a committee such that Ahmad and Salwa are included? (answer 15C4)

(iii) In how many ways can we form a committee with at least 5 females are included?

(iv) Now assume that we need to form a committee with 6 persons where order is important, i.e., we need to form a committee of this form (president, vice-president, secretary A, secretary B, secretary C, secretary D).

a. In how many ways can we form a committee such that exactly 3 females are included?(Answer: 6C3 X 7P3X10P3 !)

b. In how many ways can we form a committee such that all secretaries are females and all other positions are males? (answer 10P2 X 7C4)

c. In how many ways can we form a committee such that exactly one male is included? (answer : 6C1 X 10P1 X 7P5)

d. In how many ways can we form a committee such that exactly 2 males are included? (Answer: 6C2 X 10P2 X 7P4)

e. In how many ways can we form a committee such that exactly 4 females are included? (Note this question is the same as (iii). So 6C4 X 10P2 X 7P4 = 6C2 X 10P2 X 7P4, note that we know that 6C2 = 6C4)

In how many ways can we form a committee such that Ahmad and Salwa are included? (answer 6C2 X 2P1 X 15P4)

QUESTION 4. Fill in blank

(i) the Mickey-function $\frac{x^{0.5}+3x^{3/2}-5}{x+7}$ is $\Theta(\ \ )$ and it is $\mathbb{O}(\ \ )$

(ii) The Mickey-polynomial $\sqrt{x}\left(x^2 - x^{9/2} + 7\right)$ is $\Theta(\ \ )$ and it is $\mathbb{O}(\ \ )$

QUESTION 5. Consider the following Algorithm segment. Find the exact number of additions, multiplications, and subtractions that will be performed when the algorithm is execited. Then find the order of the Algorithm segment.

$$m = 7$$
$$For\ \ k := 4\ \ to\ \ n+1$$
$$For\ \ i := 2\ \ k+3$$
$$s = m^2 + 2 * i - k$$
$$next\ i$$
$$next\ k$$

(Note: Outer loop will be iterated (n+1) - 4 + 1 = n - 2 times. For a given k, the inner loop will be iterated (k + 3) - 2 + 1 = k+2 times. Now when k = 4, inner loop will be iterated 6 times. K = 5, inner loop will be iterated 7 times, ... , when k = n+1, the inner loop will be iterated n+ 3... Hence The Algorithm will be iterated 6 + 7 + 8 + ...+ n +3 (this is an Arithmetic sum with (n - 2) terms). Hence from class notes the sum = number of terms X (First term + last term)/2 = (n-2)(6 + n+3)/2. By staring, each iteration will perform 2 multiplication + 1 addition + 1 subtraction = 4.

Thus the exact number of additions, multiplications, and subtractions = 4(n-2)(6 + n + 3)/2 = 2(n - 2)(9 + n) = $2n^2 + 14n - 36$. Thus the order is $\Theta(n^2)$ and it is also $\mathbb{O}(x^2)$.)

QUESTION 6. Consider the following Algorithm segment. Find the exact number of additions, multiplications, and subtractions that will be performed when the algorithm is executed. Then find the order of the Algorithm segment.

$$m = 7$$
$$For\ \ k := 4\ \ to\ \ n+1$$
$$For\ \ i := 2\ \ k+3$$
$$s = m^2 + 2 * i - k$$
$$next\ i$$
$$L = k^2 + 7 * k * m - 6$$
$$next\ k$$

(note: Repeat the same as above. However, notice that each time the outer loop is iterated, there are extra 3 multiplications + one addition + one subtraction = 5.

Since the outer loop is iterated (n - 2) times, we need to add 5(n - 2) to the answer in the previous question. Thus the exact number of additions, multiplications, and subtractions = $2n^2 + 14n - 36 + 5(n-2) = 2n^2 + 19n - 46$.

QUESTION 7. Consider the following Algorithm segment. Find the exact number of additions, multiplications, and subtractions that will be performed when the algorithm is executed. Then find the order of the Algorithm segment.

$$m = 7$$
$$For\ \ k := 4\ \ to\ \ \lceil \frac{n+1}{2} \rceil$$
$$For\ \ i := 2\ \ k+3$$
$$s = m^2 + 2 * i - k$$
$$next\ i$$
$$L = k^2 + 7 * k * m - 6$$
$$next\ k$$

(Note Outer loop will be iterated $\lceil \frac{n+1}{2} \rceil - 4 + 1$. Consider two cases. n is even. Then $\lceil \frac{n+1}{2} \rceil = (n+2)/2 = n/2 + 1$. Hence outer loop will be iterated $n/2 + 1 - 4 + 1 = n/2 - 2$. Now repeat the above question using (n/2 - 2) instead of (n - 2). Second case. n is odd. Then $\lceil \frac{n+1}{2} \rceil = (n+1)/2 = n/2 + 1/2$. Hence outer loop will be iterated $n/2 + 1/2 - 4 + 1 = n/2 - 2.5$. Now repeat the above question using (n/2 - 2.5) instead of (n - 2).

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